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On the instanton-induced portion of the nucleon strangeness II: the MIT model beyond the linearized approximation

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Abstract. If instantons are introduced into the MIT bag model in such a way that the bag radii are allowed to vary, the MIT bag interior can accommodate an instanton density which is by an order of magnitude larger than in the case when the radii are fixed (although it is still significantly smaller than in the non-perturbative QCD vacuum). The instanton contribution to the baryon mass shifts is also correspondingly larger. The instanton-induced part of the scalar strangeness of the nucleon MIT bag is an order of magnitude larger than found previously, within the linearized approximation. The decrease of the model radii (which is associated with the increase of the instanton density) is very favorable from the standpoint of nuclear physics.

1 Introduction

The characteristics of non-perturbative QCD make intractable many calculations at low and intermediate energies. Effective quark models therefore retain their usefulness in numerous applications. For example, [1] used the instanton-extended version [2][1] of the MIT bag model [3, 4] in one of many studies of strangeness in nucleons [6]. However, the approach of [2] contained the so-called linearized approximation, amounting to freezing the baryon radii in their original MIT values. In the present paper we remove this approximation and calculate the effects thereof on the baryon mass splittings, and also on the nucleon strangeness results of [1]. We also explore whether this enables the resolution or alleviation of a long-standing inconsistency between the MIT bag model and nuclear physics: the standard nuclear physics descriptions employ independent nucleons, while the nucleon MIT bag radius is too large for that [7].

The first approach to consider the instanton-induced interaction within a bag model was due to Kochelev [8]. It is nevertheless important to note that he considered his own bag model [9], which is somewhat different from the MIT one. As explained in detail in [2], that line of research [9, 8, 10–14] is therefore rather different from the one in [2] and in the present paper, where we stay as close as possible to the original MIT bag model [3, 4]. The only modification with respect to the MIT model is the inclusion of the instanton-induced interaction [2]. This inclusion is necessary also *inside* the MIT bag if one allows for the non-vanishing (even if small) probability of penetration of the instanton liquid from the surrounding non-perturbative QCD vacuum, into the "perturbative" MIT bag interior. The instanton density n used in this effective instanton-induced interaction inside the bag is of course reduced with respect to the density in the non-perturbative QCD vacuum: the smaller the probability of this penetration, the larger the reduction. The reduced value of n appropriate for the MIT bag interior comes out as a result of our model calculation and fitting.

Besides defining how to incorporate instantons in the MIT bag model and finding the baryon mass shifts caused by the effective instanton-induced quark–quark interaction, [2] explored the modification of the too large value (required by the "instantonless" model fits [4]) of the strong coupling constant α_c used in the supposedly perturbative MIT bag interior. The conclusion of [2] was that the change in α_c was in the desired direction, i.e., it was reduced, but only by about 6%, which was insufficient to achieve improvement in the consistency of the perturbative description of the bag interior. Actually, it turned out that instanton effects should generally be small in the MIT bag model, since the instanton (plus anti-instanton)

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[1] In [2], the incorporation of instanton-induced interactions into the original MIT bag model [3, 4], was inspired by an analysis [5] made in the constituent quark model, which found that an effective instanton interaction led to as satisfactory a description of the mass splittings of baryons as the conventional approach using one-gluon exchange

density n appropriate for the MIT bag interior, was found [2] to be very much depleted with respect to the instanton density estimated (e.g., by [15–19]) for the true, nonperturbative QCD vacuum. The corresponding instanton-induced mass shifts were of the order of only a few MeV. However, the analysis of [2] (and, consequently, of [1]) contained an important simplifying assumption: it kept the baryon radii "frozen" in the values obtained by DeGrand et al. [4]. In this way, a full refitting of the bag model parameters (now also including the instanton density n inside the bag) was avoided. In fact, this highly non-linear problem was thereby reduced to solving the four linear equations for the four adjustable model parameters that enter the energy functional linearly: α_c, the volume-energy density B, the zero-point energy parameter Z_0, and a parameter new to the MIT bag model, namely, the instanton density n. (The quark masses were also not allowed to vary. Reference [2] adopted the quark masses of DeGrand et al. [4] in order to be able to use the results of [4] and to make a comparison with their results.) The linear equations determining the appropriate value of n and the new values of α_c, B, and Z_0 were specified by demanding that the model masses of the proton, neutron, Δ, and Ω^- be equal to the empirical masses after the inclusion of the effective instanton-induced interaction. We will call the approach of [2] the linearized approximation.

In this paper we go beyond this approximation, performing a refitting of the baryon masses which allows their radii to vary. It turns out that this leads to larger instanton densities allowed inside the MIT bags, and correspondingly to a stronger share of instantons in the energy balance of the baryon bags, accompanied by a decreased α_c, as well as by acceptable, and for nucleons even highly favorable [7,2], changes in the baryon radii. Most important for the present paper is amending the results on the nucleonic scalar strangeness obtained in [1]. On the one hand, larger instanton densities now lead to increased contributions of the instanton-induced interaction to the total scalar strangeness of the nucleon. On the other hand, the basic MIT bag strangeness (8) found by Donoghue and Nappi [20], if not far from its naive limit, may well still represent the main contribution. If it does, the total nucleon strangeness decreases with diminishing bag radii, which are in turn associated with growing instanton densities.

2 Refitting of the baryon bag parameters

Except for removing the linearized approximation, i.e., replacing it with the refitting where the bag radii are not frozen any longer, the incorporation of the instanton effects in the MIT bag follows [2] closely. The same holds also for other model details, such as the fixed model inputs, the non-strange and strange quark mass parameters ($m_u = m_d = 0$ and $m_s = 279\,\mathrm{MeV}$, respectively) and quark–antiquark ($q\bar{q}$) condensate $\langle 0|\bar{q}q|0\rangle = -(240\,\mathrm{MeV})^3$. Thus, to keep the present paper as concise as possible, we refer to [2] for all model details and parameters, and to [1] for the corresponding strangeness calculation. (For detailed technicalities of the latter, [21] may also be found helpful.)

Here we just recall that the effective instanton-induced interaction $\mathcal{L}_\mathrm{I}$, causing the instanton-induced mass shift $E_\mathrm{I}^\mathcal{B}$ of the baryon $\mathcal{B}$, is the sum of the one-, two-, and three-body terms, denoted by $\mathcal{L}_1^\mathrm{I}$, $\mathcal{L}_2^\mathrm{I}$, and $\mathcal{L}_3^\mathrm{I}$, respectively:

$$E_\mathrm{I}^\mathcal{B} = \langle\mathcal{B}| : -\mathcal{L}_\mathrm{I} : |\mathcal{B}\rangle = \langle\mathcal{B}| : -\mathcal{L}_1^\mathrm{I} - \mathcal{L}_2^\mathrm{I} - \mathcal{L}_3^\mathrm{I} : |\mathcal{B}\rangle\,, \quad (1)$$

and is defined in detail in [2]. The explicit expressions for the one- and two-body contributions ($\Delta M_\mathcal{B}^{(1)}$ and $\Delta M_\mathcal{B}^{(2)}$, respectively) are also given in [2].

Before proceeding, let us make two comments regarding our choice of the instanton-induced interaction $\mathcal{L}_\mathrm{I}$. It was derived by Nowak et al. [18] in the framework of the random instanton liquid model (RILM). They arrived at the interaction corresponding to the well-known one of Shifman, Vainshtein, and Zakharov (SVZ) [22], apart from the effects of smearing over the size of an instanton. In the limit of no smearing, it reproduces our chosen [2,1,21] local $\mathcal{L}_\mathrm{I}$, which is essentially the same as the SVZ interaction [22]. Since the SVZ interaction is induced by a single (anti-)instanton, our modeling misses multi-instanton effects. Their importance, however, was stressed in, e.g., [17, 19], putting in doubt the validity of the single-instanton approximation. The caveat is that these effects can be important when baryon bags have diameters larger than the average separation of (anti-)instantons, and this will turn out to be the presently relevant situation (since we will find instanton densities inside bags up to one third of the QCD vacuum value of $1\,\mathrm{fm}^{-4}$). Nevertheless, as discussed especially in [2], we should recall that this interaction was introduced and used [2,1,21] with the aim of capturing the intermediate-range ($\sim \frac{1}{3}\,\mathrm{fm}$) QCD effects, and the interaction we adopted is suitable for that, since the average instanton size is $\rho \approx \frac{1}{3}\,\mathrm{fm}$ [15–18,23]. Hopefully, it may capture the effects at ranges even a little beyond $\frac{1}{3}\,\mathrm{fm}$, since Nowak et al. [18] took into account the delocalization of zero modes[2]. In keeping with the basic idea of the MIT model, one assumes that really long range (i.e., confinement) effects are modeled well by the confining bag boundary.

The second comment is devoted to clarifying our inclusion of the one-body term $\mathcal{L}_1^\mathrm{I}$ into the bag model calculations of the instanton-induced contribution (1) to the baryon masses. The term $\mathcal{L}_1^\mathrm{I}$ has in fact the form of a mass term, and can be thought of as the self-energy, or the effective mass that a quark acquires from the effective interaction caused by the instanton liquid through which quarks move. Now imagine that we are working in some kind of constituent quark model where one from the start

[2] Thus, Nowak et al. [18] took into account the insights of, e.g., [17], concerning the importance of summing up a large number of interactions with different instantons. The review by Schafer and Shuryak still points out as useful their results and the RILM approach in general, observing that interactions among instantons (and hence their correlations) are important but not dominant [19]

uses effective constituent quark masses to parameterize "dressing" by non-perturbative QCD. The self-mass part of the instanton effects would in that case already be included in the constituent mass parameters. Using $\mathcal{L}_1^{\rm I}$ in the baryon mass calculation would therefore be double-counting, so in that case it must be dropped from (1). On the other hand, if we employ some approach where one uses the current, Lagrangian quark masses, like in the MIT bag model used presently and in [2,1,21], $\mathcal{L}_1^{\rm I}$ should be included in the calculation on an equal footing with $\mathcal{L}_2^{\rm I}$ and $\mathcal{L}_3^{\rm I}$. This procedure was criticized by Dorokhov [13] on the grounds that in the bag model, the role of the quark constituent mass is played by the single-quark kinetic energy eigenvalue resulting from the boundary condition confining the quarks inside the bag. According to this view, the quark non-perturbative dressing due to the $\mathcal{L}_1^{\rm I}$ part of the instanton-induced interaction would already be taken into account by the linear bag boundary condition. However, we do not accept this view because this boundary condition serves to incorporate confinement, prohibiting quark separations larger than the bag diameter scale of the order of some 2 fm, while instantons are *not* responsible for confinement [24,25] (contrary to what was thought in the early days of instanton physics). Admittedly, this argument is so far only qualitative in the sense that in the model context it is not possible to delineate precisely beyond which scale confinement effects overwhelm instanton effects. Nevertheless, the argument becomes stronger and more precise if one remembers the discussion in the previous passage: there, it was noted that the adopted instanton-induced interaction approximates well the non-perturbative QCD effects at intermediate ranges around $\frac{1}{3}$ fm, but not much further than that, and certainly not up to confinement scales of the order of the bag diameter.

As remarked above, the interaction $\mathcal{L}_{\rm I}$ is actually the same as the well-known SVZ interaction [22], including the (only seemingly different [18,26]) three-body term $\mathcal{L}_3^{\rm I}$. This term was in fact discussed in [2] because, at that point, it was not clear whether the contribution of $\mathcal{L}_3^{\rm I}$ vanished for Λ, as it did for other baryons. Therefore, [2] avoided the need to compute the contribution from the complicated-looking $\mathcal{L}_3^{\rm I}$ by showing that it could contribute only to the mass of the Λ and by omitting the Λ from the analysis. However, it turns out that the mass shift due to the three-body interaction, if non-zero, must be small for the Λ [27]. (In an explicit evaluation one can see that all terms in the $\mathcal{L}_3^{\rm I}$-contribution would cancel in the SU(3)-symmetric limit. This contribution slightly differs from zero only because the strange-quark wave functions differ somewhat from the non-strange ones.) Neglecting therefore this contribution to M_Λ, the total instanton-induced mass shift (1) consists of one- and two-body contributions only [2]:

$$E_{\rm I}^{\mathcal{B}} = \langle \mathcal{B}| : -\mathcal{L}_1^{\rm I} - \mathcal{L}_2^{\rm I} : |\mathcal{B}\rangle \equiv \Delta M_{\mathcal{B}}^{(1)} + \Delta M_{\mathcal{B}}^{(2)}, \qquad (2)$$

for all baryons $\mathcal{B}$, including the Λ. There is hence no need to drop the Λ from the analysis, so in this respect, this calculation is slightly more complete than in [2]. However, when we did drop the Λ from the present fit in order to check the effects thereof, the results were affected very little.

Therefore, the only significant difference in modeling with respect to [2] is that in the present paper we want to perform a full refitting of the model parameters, including the variation of the bag radii. Maybe some reader might then object that for each baryon $\mathcal{B}$, its bag radius would become a new free parameter, and the number of fitting parameters would become larger than the number of experimental baryon masses $M_{\rm exp}^{\mathcal{B}}$ to be fitted. Fortunately, this is not so, because each radius $R_{\mathcal{B}}$ of a bag in equilibrium must satisfy the pressure-balance condition. That is, the equilibrium bag radius $R_{\mathcal{B}}$ of the baryon $\mathcal{B}$ is fixed by minimizing the bag model mass $M_{\rm bag}^{\mathcal{B}}$,

$$\frac{{\rm d}M_{\rm bag}^{\mathcal{B}}}{{\rm d}R_{\mathcal{B}}} = 0 \qquad (\mathcal{B} = N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega)\,, \qquad (3)$$

and is not a free, adjustable parameter like Z_0, α_c, B and n.

The MIT bag energy functional $M_{\rm bag}^{\mathcal{B}}$ of the baryon $\mathcal{B}$ now depends also on the instanton density n, because

$$M_{\rm bag}^{\mathcal{B}}[R_{\mathcal{B}}, Z_0, \alpha_c, B, n] = E_Q^{\mathcal{B}} + E_0^{\mathcal{B}} + E_M^{\mathcal{B}} + E_E^{\mathcal{B}} + E_V^{\mathcal{B}} + E_{\rm I}^{\mathcal{B}} \qquad (4)$$

now contains the instanton contribution $E_{\rm I}^{\mathcal{B}}$ (1), in addition to the kinetic energy of the confined quarks $E_Q^{\mathcal{B}}$, the zero-point energy $E_0^{\mathcal{B}}$, the color magnetic energy $E_M^{\mathcal{B}}$, the color electric energy $E_E^{\mathcal{B}}$, and the volume energy $E_V^{\mathcal{B}}$. The expressions for these five latter contributions are given in [4]. (In (4), the dependence of $M_{\rm bag}^{\mathcal{B}}$ on the quark mass parameters m_u, m_d, m_s and the condensate $\langle 0|\bar{q}q|0\rangle$ is not indicated, as they are not adjustable parameters but fixed model inputs.)

In the circumstances explained above, the most practical and numerically tractable way to perform the model fit to the empirical baryon masses, is to pose it as the problem of minimization of the positive definite functional F:

$$F[\{R_{\mathcal{B}}\}, Z_0, \alpha_c, B, n] \equiv F_M + F_R\,, \qquad (5)$$

$$F_M \equiv \sum_{\mathcal{B}} (M_{\rm exp}^{\mathcal{B}} - M_{\rm bag}^{\mathcal{B}})^2\,, \qquad (6)$$

$$F_R \equiv \sum_{\mathcal{B}} \frac{1}{\mathcal{M}^2} \left(\frac{dM_{\rm bag}^{\mathcal{B}}}{dR_{\mathcal{B}}}\right)^2, \qquad (7)$$

where both sums run over the baryons $\mathcal{B}$ in the ground-state octet $(N, \Lambda, \Sigma, \Xi)$ and decuplet $(\Delta, \Sigma^*, \Xi^*, \Omega)$. Thus, note that in the present fitting procedure all baryon masses enter on an equal footing, whereas [2], similarly to [4], chooses some masses somewhat arbitrarily to fix the parameters and predict the other masses.

In the functional F, the first sum, F_M, represents the deviation of the bag model masses $M_{\rm bag}^{\mathcal{B}}$ from the experimental baryon masses $M_{\rm exp}^{\mathcal{B}}$. The second sum, F_R, is a measure of the deviation from the situation of the perfectly satisfied pressure-balance condition. The role of the constant $\mathcal{M}$ is just to ensure that both terms have the same dimension, and we choose the typical baryonic mass scale of 1 GeV to fix its value: $\mathcal{M} = 1\,\mathrm{GeV}$. (Of course, there is some arbitrariness in the choice of the functional F; for example, we could replace $\mathcal{M}$ by $M^{\mathcal{B}}_{\mathrm{exp}}$ in each term of the sum F_R. However, we have checked that varying the scale $\mathcal{M}$ does not influence our results significantly, giving us confidence that this arbitrariness is not a problem in practice.)

The functionals defined by (5)–(7), namely F, F_M and F_R, all depend on the model parameters Z_0, α_c, B, n and on the set of the bag radii $\{R_{\mathcal{B}}\}$ of the octet and decuplet baryons. The strict approach to the model fitting through the functional minimization would be to pick the initial values of the free parameters, $Z_0^{(0)}, \alpha_c^{(0)}, B^{(0)}, n^{(0)}$, and find the equilibrium radii $R_{\mathcal{B}}$ by minimizing the functional F_R to, ideally, $F_R = 0$, where the conditions (3) would be strictly satisfied. Then the functional F_M should be calculated. This two-step process should be repeated with varied values of the free parameters over and over again by some minimization routine (for example based on simplex minimization) till F_M is as close to zero as possible. However, this two-step process, where F_R would be minimized before each call to F_M, is computationally rather intractable in practice. Fortunately, it turns out that for the degree of accuracy that is sensible to demand from the MIT bag model (set by F_M of the original fit [4]), it is sufficient to perform the refitting by varying simultaneously Z_0, α_c, B, n and the set $\{R_{\mathcal{B}}\}$ to minimize the joint functional F. Nevertheless, one should accept only those minimizations where F_M is the overwhelming share, and F_R only a small part of $F = F_M + F_R$; otherwise the fit to the experimental masses would be done away from the equilibrium bag radii. Ideally, the aim would be $F = 0$, but since it is not possible to model all experimental masses *exactly*, one should look for such parameter values for which F is sufficiently small. In the present model it is sensible to demand $F < 3\times10^{-3}\,\mathrm{GeV}^2$, since the original MIT bag fit [4] gives $F_M = 3.2 \times 10^{-3}\,\mathrm{GeV}^2$.

The minimization of F by the simplex method [28], which had already been proved as robust and reliable in earlier applications [29–32], has turned out to be very suitable also in the present case.

3 Results and discussion

It is necessary to give some thought as to which outputs of the minimization procedure can be accepted as solutions to our problem. The present situation is different than in [2], where the frozen radius approximation reduced the problem to solving a set of linear equations, so that the solution was unique once we chose which baryon masses would be used to fix the parameters. In the present case, the functional minimization finds many local minima of the functional (5). In which of them the minimization will end up depends on which part of the parameter space one starts from. Moreover, many of these minima can be acceptable in the sense of a sufficiently small value of the minimized functional F. We thus face the problem of non-uniqueness of the solutions. Fortunately, the smallness of the minimized functional F is not the only criterion; clearly, a fit resulting in a good mass spectrum would anyway be unacceptable if it also resulted in physically unacceptable values of the bag radii or fitting parameters. This must always be kept in mind, as the problem was mathematically posed in such a way that it is possible to get an excellent fit to the masses, but with the bag radii and parameters devoid of any physical justification.

3.1 Practically instantonless bag

The first thing to check is the limit of the vanishing instanton density n inside the bag. This is basically the case of the pure MIT bag model except that our model fitting is done by minimizing the functional (5). This is different from the original MIT bag fitting procedure [4] where the parameters B, Z_0 and α_c were fixed by singling out three hadrons and constraining their model masses to be the experimental ones. From the model standpoint, it is very satisfying that for $n = 0$ inside the bag, our different fitting procedure leads to the description of baryons very similar to the original MIT bag fitting procedure [4]. In addition to that, we note that when we depart from the limit of vanishing instanton density inside the bag and finally allow $n \neq 0$, the minimization of the functional (5) leads, among various outcomes, also to several solutions where the values of n are non-vanishing but extremely small, $n \lesssim 10^{-6}\,\mathrm{GeV}^4$. This is practically negligible in comparison with the density $n_0 \approx 1.6 \times 10^{-3}\,\mathrm{GeV}^4 = 1\,\mathrm{fm}^{-4}$ estimated reliably (e.g., see [15–18,23]) for the non-perturbative QCD vacuum outside the bag. The resulting baryon masses and radii (as well as values of the variable model parameters) are very close to each other in all these cases of very small n, and also very similar to the pure MIT case ($n = 0$), as one would expect, although all those cases are formally different solutions. It is thus clear that they all describe very similarly ("practically uniquely") the situations when $n \to 0$. This shows that non-uniqueness of the solutions is not a problem in practice, and the same happens in the more interesting cases with significant values of n, discussed in the next subsection.

3.2 Appreciable instanton density inside the bag

Let us now discuss the first major interest of this paper: the cases when instanton densities n inside the bag are significantly different from zero. Indeed, in most cases we obtained interesting solutions where the densities n inside the bag are an order of magnitude larger than in the linearized approximation [2], where[3] $n = 0.266 \cdot 10^{-4}\,\mathrm{GeV}^4$. However, for all acceptable minimizations of the functional (5), we find that they are still appreciably lower (at least

[3] The dimensionless density $\widetilde{n}$ used in [2] and n are related by $n \equiv \widetilde{n}\rho^{-4}$, where ρ is the average instanton radius. Throughout, we have adopted the standard value $\rho = 1/600\,\mathrm{MeV}^{-1} \approx 1/3\,\mathrm{fm}$ (e.g., see [15–18,23])

Table 1. The fit for the input quark masses $m_u = m_d = 0$, $m_s = 279\,\mathrm{MeV}$ and the quark-antiquark vacuum condensate $\langle 0|\bar{q}q|0\rangle = -(240\,\mathrm{MeV})^3$. We display the separate energies $E_X^{\mathcal{B}}$ $(X = 0, V, Q, M, E, I)$ contributing to $M_{\mathrm{bag}}^{\mathcal{B}}$, the mass of the baryon bag, to be compared with the corresponding experimental baryon mass $M_{\mathrm{exp}}^{\mathcal{B}}$ in the first column. The output values of the bag model parameters B, Z_0, α_c and n are given in the lowest part of Table 1. All the masses and energies are given in GeV, and the bag radii $R_{\mathcal{B}}$ in inverse GeV, while Z_0 and α_c are dimensionless

Baryon $\mathcal{B}$	$M_{\mathrm{exp}}^{\mathcal{B}}$	$M_{\mathrm{bag}}^{\mathcal{B}}$	$R_{\mathcal{B}}$	$E_0^{\mathcal{B}}$	$E_V^{\mathcal{B}}$	$E_Q^{\mathcal{B}}$	$E_M^{\mathcal{B}}$	$E_E^{\mathcal{B}}$	$E_I^{\mathcal{B}}$
N	0.938	0.959	4.365	−0.526	0.176	1.404	−0.128	0.000	0.033
Λ	1.116	1.120	4.383	−0.524	0.178	1.557	−0.127	0.003	0.033
Σ^+	1.189	1.172	4.529	−0.507	0.197	1.513	−0.094	0.003	0.061
Ξ^0	1.315	1.306	4.475	−0.513	0.190	1.688	−0.109	0.003	0.047
Δ	1.232	1.248	5.130	−0.448	0.286	1.195	0.109	0.000	0.107
Σ^*	1.385	1.388	5.073	−0.453	0.277	1.370	0.096	0.003	0.095
Ξ^*	1.533	1.526	5.027	−0.457	0.269	1.543	0.084	0.003	0.083
Ω^-	1.672	1.661	4.978	−0.461	0.261	1.716	0.074	0.000	0.071
$B^{1/4} = 0.150\,\mathrm{GeV}$		$Z_0 = 2.296$		$\alpha_c = 0.394$		$n = 0.512 \cdot 10^{-3}\,\mathrm{GeV}^4$			

by the factor of 3 or more) than the non-perturbative vacuum density $n_0 \approx 1\,\mathrm{fm}^{-4} = 1.6 \cdot 10^{-3}\,\mathrm{GeV}^4$. Therefore, we do not get a description of baryons which would be drastically different from the original MIT bag one [4], but we do obtain the desirable decrease of α_c which is noticeably stronger than the corresponding decrease obtained earlier in the linearized approximation [2]. (In the case depicted in Table 1, α_c is by 30% smaller than in [4].) In the solutions with decreased α_c, we also observe the decrease of the baryon radii $\{R_{\mathcal{B}}\}$. As mentioned above, this is very desirable from the standpoint of nuclear physics, as explained by, e.g., Brown et al. [7]. Namely, standard nuclear physics descriptions favor the picture of nuclei as made of *independent* nucleons interacting by effective boson exchange, but the empirical sizes of nuclei indicate that the "standard" MIT nucleon bags with $R_N \approx 1\,\mathrm{fm}$ are already somewhat too large [7] for that. For this reason, we give in Table 1 the case with the smallest nucleon radius for which we managed to achieve an acceptable fit. Other physically acceptable solutions have somewhat smaller n and somewhat larger radii. Table 2 gives a kind of condensed overview of several representative fits; e.g., the last line in Table 2 summarizes Table 1, the case with the highest n which leads to a fit acceptable by all criteria. The general features of the acceptable fits are the following:

(a) The values of the functional F are around 1.3 to $1.2 \times 10^{-3}\,\mathrm{GeV}^2$ (out of which only less than a percent is F_R). This gives the rough limit on the accuracy of reproduction of the mass spectrum within the present model. The average deviation from an experimental baryon mass is 11 MeV. In fact, the predictions for the masses of N and Σ are the worst. They are too high for N and too low for Σ by some 20 MeV. The other masses are within 10 MeV from the experimental masses. (In the MIT fit [4], the N mass belongs to those constrained to experimental values to fix the model parameters, but then the Σ mass is too low by 45 MeV.) Overall, our approach to fitting of baryon masses gives noticeably smaller sum of squared deviations from the empirical baryon masses, F_M, than the original MIT bag fit [4] and the linearized approximation (where $F_M = 3.5 \times 10^{-3}\,\mathrm{GeV}^2$) [2].

Table 2. Brief overview of some typical fits. (The fit given in Table 1 is one example of them.) The values of functional F in the last column show the good quality of the fits. The interdependence of the adjustable bag parameters (n, α_c, B, Z_0) and the bag radii is summarily depicted utilizing the average octet and decuplet radii, $\overline{R}^O$ and $\overline{R}^D$, respectively

$n \times 10^3$ [GeV^4]	α_c	$B \times 10^4$ [GeV^4]	Z_0	$\overline{R}^O$ [GeV^{-1}]	$\overline{R}^D$ [GeV^{-1}]	$F \times 10^3$ [GeV^2]
0.290	0.485	4.031	1.865	5.0	5.6	1.14
0.310	0.474	4.188	1.930	4.9	5.4	1.18
0.398	0.437	4.612	2.114	4.7	5.2	1.26
0.512	0.394	5.058	2.296	4.4	5.1	1.29

(b) Going beyond the linearized approximation and thereby allowing the bag radii to vary leads to some significant changes with respect to the results in linearized approximation. Notably, Table 1 shows the instanton contributions to baryon energies are an order of magnitude larger than in the linearized approximation [2]. Such instanton contributions are present not only in Table 1, but in a large majority of the fits, since the instanton densities in most of the presently obtained solutions are an order of magnitude larger than n obtained in the linearized approximation [2]. Nevertheless, since the instanton contributions to the bag masses are still much smaller than other contributions (except $E_E^{\mathcal{B}}$), the general picture of baryons is not drastically altered with respect to the original MIT bag phenomenology [4].

(c) In most cases this relatively large n inside the bag leads to the decrease of α_c, although there are also some fits with relatively large n where α_c grows back close to its MIT value [4]. Then, however, such a larger α_c is also accompanied by an excessive increase of the bag radii. In particular, this yields a nucleon radius even larger than

Table 3. Dependence of the scalar strangeness of the nucleon on the instanton density n, or on the bag radius R_N associated with this density. For comparison, we recall that in the linearized approximation [1] the instanton-induced strangeness from $\mathcal{L}_1^{\rm I}$ was $\langle N|\bar{s}s|N\rangle_{\mathcal{L}_1^{\rm I}} = 0.035$, whereas the contribution from $\mathcal{L}_2^{\rm I}$ was $\langle N|\bar{s}s|N\rangle_{\mathcal{L}_2^{\rm I}} = 0.023$ (at $R_N = 5.00\,\mathrm{GeV}^{-1} \approx 1\,\mathrm{fm}$). In the last column, the choice $\eta = 0$ maximizes the basic bag strangeness contribution of [20]; this is nevertheless only the so-called naive bag model limit, and in fact η remains undetermined. The fixed model inputs (m_u, m_d, m_s and $\langle 0|\bar{q}q|0\rangle$) are the same as in Table 1 and Table 2, and are discussed in detail in the main text

$n \times 10^3$ [GeV^4]	R_N [GeV^{-1}]	$\langle N\|\bar{s}s\|N\rangle_{\mathcal{L}_1^{\rm I}}$	$\langle N\|\bar{s}s\|N\rangle_{\mathcal{L}_2^{\rm I}}$	$\langle N\|\bar{s}s\|N\rangle_{\mathcal{L}^{\rm I}}$	$\frac{\langle N\|\bar{s}s\|N\rangle_{\rm basic}}{(1-\eta)}$
0.290	4.994	0.22	0.09	0.31	7.21
0.310	4.909	0.24	0.10	0.34	6.85
0.398	4.658	0.29	0.15	0.44	5.85
0.512	4.365	0.36	0.22	0.58	4.82

in the MIT case [4], so that such solutions must be discarded as unacceptable from the point of view of nuclear physics as explained above. The interdependence of the model parameters and the baryon bag radii which minimize F is such that α_c decreases while Z_0 and B increase with the decrease of the bag radii. This is illustrated in Table 2, which, for four different fits, displays the *average* octet (O) and decuplet (D) radii, $\overline{R}^O$ and $\overline{R}^D$, for four different fits. The notion of the average multiplet radii $\overline{R}^O$ and $\overline{R}^D$ is useful since the octet baryon radii are similar to each other, and the decuplet baryon radii are similar to each other. The decuplet radii are also some 10% larger than the radii of the octet baryons.

3.3 Instanton-induced strangeness inside the bag

Inspection of [1] easily shows that going beyond the linearized approximation, and the above effects thereof, does not change the results of [1] on the vector, axial-vector and pseudo-scalar strangeness of the nucleon bag: the instanton-induced contributions to them are still vanishing.

In contrast to that, the instanton-induced scalar strangeness is enhanced an order of magnitude over what it was in the linearized approximation [2], following the increase of the instanton density n. This is seen in Table 3, which shows the dependence on the instanton density, or on the bag radius associated with this density, of various scalar strangeness components of the nucleon. The instanton-induced contributions due to $\mathcal{L}_1^{\rm I}$ and $\mathcal{L}_2^{\rm I}$, respectively denoted by $\langle N|\bar{s}s|N\rangle_{\mathcal{L}_1^{\rm I}}$ and $\langle N|\bar{s}s|N\rangle_{\mathcal{L}_2^{\rm I}}$, comprise the overwhelming share of the total instanton-induced contribution $\langle N|\bar{s}s|N\rangle_{\mathcal{L}^{\rm I}}$. We do not display $\langle N|\bar{s}s|N\rangle_{\mathcal{L}_3^{\rm I}}$, the contribution due to $\mathcal{L}_3^{\rm I}$, as it contributes only to the third decimal place.

Although our present interests are the instanton-induced contributions, we should also comment on the basic strangeness of the nucleon MIT bag, $\langle N|\bar{s}s|N\rangle_{\rm basic}$ (found by Donoghue and Nappi [20]),

$$\langle N|\bar{s}s|N\rangle_{\rm basic} \equiv (\eta - 1)\langle 0|\bar{q}q|0\rangle \, \frac{4\pi}{3} \, R_N^3 \,. \tag{8}$$

It is the product of the nucleon bag volume $V_N = (4\pi/3) R_N^3$ and $\langle 0|\bar{q}q|0\rangle$, the expectation value of the $\bar{q}q$ scalar condensate in the true, non-perturbative QCD vacuum, but also of the factor $\eta - 1$ which has unfortunately remained quantitatively undetermined. Its determination is beyond the scope of the present paper. Let us just quote [20] that η ($0 < \eta < 1$) is in general some decreasing function of the bag radius, since $R_N \to \infty$ corresponds to $\eta \to 0$. The case $\eta = 0$ is called the naive bag model limit and obviously maximizes the basic bag strangeness (8). This limit was, for definiteness, the only case of the basic bag strangeness $\langle N|\bar{s}s|N\rangle_{\rm basic}$ considered in [1]. Although in the present paper even the $\eta = 0$ limit of $\langle N|\bar{s}s|N\rangle_{\rm basic}$ is not so much larger than the (now increased) quantity $\langle N|\bar{s}s|N\rangle_{\mathcal{L}^{\rm I}}$ as was the case in [1], it is still larger by an order of magnitude for all radii displayed in Table 3. The most widely accepted value of the condensate, adopted also in [2] and the present paper, $\langle 0|\bar{q}q|0\rangle = -(240\,\mathrm{MeV})^3$, leads to $\langle N|\bar{s}s|N\rangle_{\rm basic}$ considerably exceeding the empirical value of the total scalar strangeness (determined by, e.g., [33], from the σ-term estimated from the πN scattering data and the masses of Ξ, Σ and Λ),

$$\langle N|\bar{s}s|N\rangle \approx 2.8 \,. \tag{9}$$

Of course, lower values of $\langle 0|\bar{q}q|0\rangle$ trivially decrease $\langle N|\bar{s}s|N\rangle_{\rm basic}$. For example, the choice $\langle 0|\bar{q}q|0\rangle = -(200\,\mathrm{MeV})^3$, as in [1], amounts to reducing the values of $\langle N|\bar{s}s|N\rangle_{\rm basic}/(1-\eta)$ in Table 3 by the factor $(200/240)^3 = 1/1.728$, but this still gives rather large values. Instantons inside the bag help with that. Admittedly, (8) shows clearly that $\langle N|\bar{s}s|N\rangle_{\rm basic}$ is not directly dependent on instantons and their density n inside the bag, but there is an indirect connection: first, the volume factor in (8) decreases with the radii as R_N^3, and diminishing radii are associated with increasing n. Second, $1 - \eta$ *also* falls with R_N. Thus, even if $\langle N|\bar{s}s|N\rangle_{\rm basic}$ in the original, instantonless MIT bag model would be too close to its (too large) naive value, this potential problem would now be alleviated (more strongly than R_N^3) by lower values of R_N, occurring at higher n.

As already stressed, the most interesting effect of the presently increased values of the instanton density n is the considerable enhancement of the instanton-induced scalar strangeness, and we would like to point out that this enhancement is *not* due to a favorable choice of the fixed input parameters m_u, m_d, m_s and $\langle 0|\bar{q}q|0\rangle$. In fact, our adoption of the fixed input parameters of [2] and [4] was motivated not only by the ease of comparison with these papers. This choice is also suitable for stressing that the present enhancement (of the *instanton-induced* strangeness) is not an effect of the choice of the model parameters. This is because the values of the quark mass parameters and of the vacuum quark–antiquark ($\bar{q}q$) scalar condensate used in [2] and in obtaining all presently displayed results ($m_u = m_d = 0$, $m_s = 279\,\mathrm{MeV}$, $\langle 0|\bar{q}q|0\rangle = -(240\,\mathrm{MeV})^3$), actually lead to a smaller instanton-induced scalar strangeness than those used in [1] ($m_u = m_d = 8\,\mathrm{MeV}$, $m_s = 200\,\mathrm{MeV}$, $\langle 0|\bar{q}q|0\rangle = -(200\,\mathrm{MeV})^3$). That the latter choice [1] of these inputs gives (at a given instanton density n) an even more enhanced instanton-induced scalar strangeness than that in Table 3 is most easily understood if one notes the role of the characteristic pre-factors, denoted by $\mathcal{F}_f$ in [2,1], appearing in the instanton-induced interaction $\mathcal{L}_\mathrm{I}$.

The factor $\mathcal{F}_f$ pertaining to a flavor f ($f = u, d, s$), is composed of the corresponding quark mass parameter m_f, the average instanton size $\rho \simeq \frac{1}{3}\,\mathrm{fm}$ [15–18,23], and the $\bar{q}q$ condensate $\langle 0|\bar{q}q|0\rangle$, in the following way:

$$\mathcal{F}_f \equiv \frac{1}{m_f\rho - \frac{2\pi^2}{3}\rho^3\langle 0|\bar{q}q|0\rangle}\,,\ (f = u, d, s). \tag{10}$$

Obviously, smaller values of m_u, m_d, m_s and $\langle 0|\bar{q}q|0\rangle$ will increase the $\mathcal{F}_f$, and vice versa.

Let us consider the concrete sets of input parameters, those of [2,1]. Changing $m_u = m_d$ from 0 to 8 MeV actually does not have a significant influence on the instanton-induced strangeness, since m_u and m_d are anyway small at the hadronic mass scale (where 8 MeV can be approximated by 0). Nevertheless, the decrease of m_s from 279 MeV to 200 MeV is quite important for further increasing the instanton-induced strangeness significantly over the values in Table 3. In fact, the effect thereof is comparable to the effect of the decrease of $|\langle 0|\bar{q}q|0\rangle|$ from $(240\,\mathrm{MeV})^3$ to $(200\,\mathrm{MeV})^3$.

Besides the effect on the $\mathcal{F}_f$, the change of quark masses changes the quark wave functions, and, more importantly, the quark energy denominators appearing in the calculation of the nucleon strangeness (as can be seen in [1]). This way, the decrease of m_s from 279 MeV to 200 MeV still further increases the instanton-induced strangeness. (Again, the increase of $m_u = m_d$ from 0 to 8 MeV is too small to have a significant influence.) The effect of the quark energy denominators and wave functions is not so clearly disentangled as the effect of the $\mathcal{F}_f$-factors, so an explicit calculation is needed to show that the effect is of comparable magnitude. However, the important thing for the present discussion is that this effect changes the nucleon strangeness in the same direction as the $\mathcal{F}_f$-factors.

4 Conclusion

To summarize, we first remark that we did not perform a "first-principle"-type calculation of the probability of penetration of the instanton liquid from the surrounding non-perturbative QCD vacuum into the bag. Rather, we performed model fits to the baryon masses and these fits showed which values of the instanton densities can be accommodated inside the MIT bag (in a physically acceptable way) and what the effects thereof would be. In the present paper, we went beyond the linearized approximation of [2], and the bag radii were allowed to vary in the course of parameter fitting, which was performed so that the radii had to satisfy the pressure-balance condition. In this approach, the importance of the instanton-induced interaction allowed to act inside the quark bag is increased in every way, and not only for the baryon mass shifts, the size of which follows the increase of the instanton density inside the bag. We namely found that the MIT bag interior can accommodate instanton densities an order of magnitude larger than found in the linearized approximation [2]. They grow faster than the inverse of the bag volume with decreasing bag radii returned by the fitting procedure. The growth of the instanton-induced scalar strangeness of the nucleon is even slightly faster than that when the nucleon radius falls. The instanton-induced part of the nucleon strangeness is now an order of magnitude larger than the instanton-induced strangeness found in the linearized approximation [1]. The quantity (8), considered as the basic MIT bag contribution to the nucleon strangeness [20], remains undetermined also in the present work, but we could show that it must be smaller in the instanton-enriched MIT bag than in the original MIT bag. This is good, because this quantity alone has the potential to overshoot strongly the empirical value (9) of $\langle N|\bar{s}s|N\rangle$. Also, it turns out that allowing for the possibility of instanton densities significantly different from zero inside the MIT bags now enables the favorable up to 30% decrease of α_c. More importantly, it also enables the decrease of the nucleon MIT bag radius by more than 10%, improving somewhat the consistency of the MIT bag model with nuclear physics.

After so summarizing the concrete results of this paper, we close by making a more speculative comment on how our enabling substantial instanton liquid densities inside the MIT bag seems to improve the consistency of the model. Let us first note that, contrary to some earlier statements [34], Schafer [35,36] has recently shown that the instanton liquid model, including the one we use, is not necessarily in conflict with the expansion in large N_c, the number of QCD colors. Then we recall an observation of Bardeen and Zakharov [37] concerning the N_c scaling of the bag constant B. Inside hadrons, which can be modeled by the bag model, the quark color fields most probably suppress instantons and other non-perturbative fluctuations. However, a smooth large-N_c limit seems to indicate [37] that the suppression of these fluctuations, including instantons, is not very strong (in keeping with relatively low values of B coming from phenomenological fits). Our modification of the bag, containing considerable instanton liquid densities inside, is obviously more consistent with the Bardeen and Zakharov result [37] than the original MIT model, where this suppression is complete.

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